The 14th International Young Physicists Tournament
The winning report

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Introduction

In one of the recent volumes of *Physics competitions* was published an overview article about the IYPT, held in the end of May, 2001 in Finland. The reader could obtain a general impression about rules of the game and also some problems were touched.
Here we want to present one of the solutions, the final and wining solution of the Slovak team, in a more detailed way. The aim is to show, how a rather complex problem could be prepared and presented by high schools students in a (at least for the jury) satisfactory form.

Task

One end of a thread is immersed in a vessel filled with water. The other end hangs down outside without contact with the outer wall of the vessel. Under certain conditions, one can observe drops on that end of the thread. What are those conditions? Determine how the time of appearance of the first drop depends on relevant parameters.

Solution - induction

First of all we have to discuss the problem itself. There are two basic questions:

1. What are the conditions of the appearance of the first drop
2. How does the time of the first drop depends on *relevant parameters*

So, first of all, we should try to guess the parameters and to describe them. In general, it is possible to divide them in to three main groups:

- Parameters concerning the set-up

Here are three main quantities that can strongly influence the time. It is **x**, the vertical distance between water surface and the bending point of the thread in the top of the glass. This point will be in the future addressed as the *critical point*. On the outer side of the glass the thread is hanging down with the height **h**, and the total length of the thread is **L**. It is worth to mention, that the intuitive equation **L=x+h** is not valid, because the vessel can be sloped. The correct equation joining these 3 parameters could be $L = \frac{x}{\cos(\alpha)} + h$ with α the bending angle.

Of course, also some other parameters like the thickness of the vessel could influence the time, but we consider them as minor important.

- Parameters concerning the thread

In this case the exact enumeration of parameters is fairly not so easy as in the set-up case. As the only exact parameter we consider **R** as the radius of the thread. Furthermore, the material of the thread is very important for us, especially if we handle hydrophobic (cotton) or hydrophilic (synthetic) materials. Also the structure is of big importance. In our model, we consider the thread as a set of parallel, independent capillaries. For this kind of model, effective relevant parameters are the mean number of capillaries per a cross surface of the thread **n**/mm^2 and a mean radius of the capillary **r**. The model will be discussed later in more detail.

- Other parameters

In this section we have to consider parameters connected with the water (salinity, temperature) and the environment parameters (temperature, humidity). These parameters will be discussed only in qualitative way.

Model

As mentioned in the previous section, we had to build up a model for solving this task. The reason is very simple – real thread is far to complex to count with it, as it is. So, for the part of the problem concerning the flow of the water through the thread, we imagine the thread as a system of parallel capillaries. These capillaries are independent and the total flow through the thread is given by the sum of individual flows. An equation holds:

$$q\pi R^2 = \pi N r^2 \quad (1)$$

where **q** is the ratio of free space in the thread and **N** is the total number of capillaries in the thread. For the flow, we use the Poiseuille's equation

$$Q = \frac{\pi}{8\eta} r^4 \frac{\Delta p}{L} \quad (2)$$

for one capillary.

By concerning the evaporation, we imagine a simple smooth wet surface of the thread.

Other things like the vessel, water and environment are also idealized by neglecting the friction etc.

The main idea is, that the water flows through the capillaries in the thread. Up to the critical point, the gravitational force is acting against the capillary force. After reaching the point, the gravitational force is helping to suck the water from the vessel. The next important point is the level of the water. After reaching it, the water is sucked down sufficient by the gravitational force (so called wine-effect, this method is used by sucking wine from the barrel without the sediments), still with help of the capillary. At the end of the thread, the capillary effect is working against the gravitational force, BUT… This is probably the main goal here that by sucking of the water the inner radius of the capillary **r** is relevant, whereas by dropping on the end the radius of the whole thread **R** is relevant. And since **R>>r**, the capillary effect on the end of the thread is not so important as during the first phase of sucking.

We have shown in the experiments, that also evaporation is very important phenomenon in this problem. The time of appearance of the first drop is counted in tenths of minutes, and during this time a significant amount of water can be evaporated from the thread. We introduce an effective parameter of the evaporation $\mathbf{C_{evap}}$ and state that the "flow" of evaporation is given only by this parameter and the surface of the thread, linearly. The condition for appearing of the drop is then rather simple, the flow of the water must be bigger than the evaporation flow.

Wet thread

The task is not strict by specifying, if the thread in the beginning is wet or dry. So, we have to consider both possibilities. We begin with the wet thread and the reason is simple. The case of dry thread is only combination of two sequential problems, the wetting up of the thread and the problem of wet thread itself.

Now, finally, we present here the solution. Since the flow is constant, we can express it independently of the time. The flow through **N** capillaries is

$$Q_N = NQ_1 = \frac{\pi}{8\eta}\frac{q^2R^4}{N}\frac{\left((h-x)\rho g - \frac{2\sigma}{R}\right)}{L} \quad (3)$$

where already the equation (1) was used. The calculation of the time of appearance of the first (and every next) drop, by NEGLECTING the evaporation, is very simple

$$T_{drop} = \frac{8\eta N V_{drop}}{\pi q^2 R^4}\frac{L}{\left((h-x)\rho g - \frac{2\sigma}{R}\right)} \quad (4)$$

where $\mathbf{V_{drop}}$ can be calculated approximately by $\frac{2}{3}\pi R^3$. We immediately see our first condition from (4), since the time has to be positive. This is

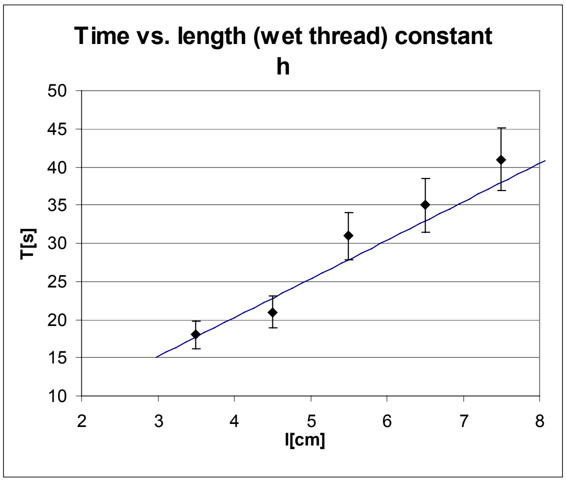
$$h - x > \frac{2\sigma}{\rho g R}. \ (5)$$

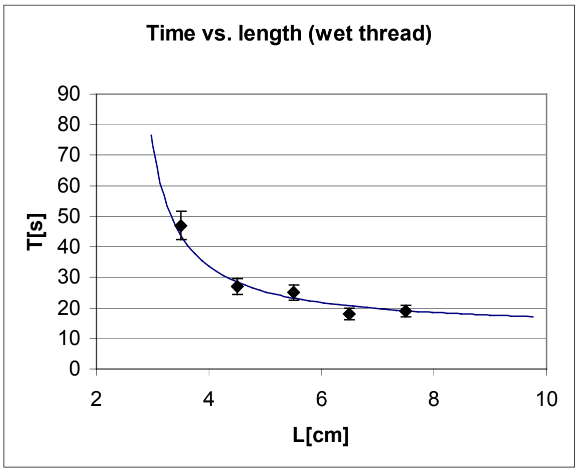
After some reflection we can bind (5) with a simple physical phenomena. The gravitational force has to be big enough to bring the drop to fall.
It is possible also to consider the evaporation. We can measure the coefficient $\mathbf{C_{evap}}$ by hanging the thread and let it dry, measuring the amount the water and the time. After that, we can write:

$$Q_{Vap} = 2\pi RL.C_{evap} \ (6)$$

$$T_{drop} = \frac{V_{drop}}{(Q_N - Q_{Vap})} \ (7)$$

what gives the simple condition, that the water flow has to be bigger than the evaporation flow. Experimental results confirm good precision of the model and calculation.





Dry thread

As mentioned, the problem of a dry thread is more complex than the previous one. First, we have to consider the problem of getting the thread wet.
In this part, the flow is not constant during the time. We define a new variable **l** meaning the length of the thread, which is already wet. Naturally, it holds **0≤l≤L**. Now the flow and also the evaporation depend on this length.
Here again we first neglect the evaporation to get the results more clear. We also divide the motion of the waterfront in the thread in to two parts.
First, there is the motion up, up to the critical point. The flow can be expressed:

$$Q_N^{(1)} = \frac{\pi}{8\eta} \frac{q^2 R^4}{N} \frac{\frac{2\sigma}{r} - l\rho g}{l} \quad (8)$$

We use a differential equation, since the flow depends on the length **l**:

$$Q_N^{(1)}(l)\mathbf{d}t = q\pi R^2 \mathbf{d}l \ \ (9)$$

and get the result for the time

$$t_1 = \frac{8\eta N}{q^2 R^2 g^2 \rho^2}\left[-\frac{2\sigma}{r}\ln\left(1 - \frac{g\rho x r}{2\sigma}\right) - g\rho x\right] (10)$$

This rather complex equation gives us another condition on the parameters. The argument in the logarithm must be positive, meaning

$$\frac{g\rho xr}{2\sigma} < 1 \quad \Rightarrow \quad x < \frac{2\sigma}{g\rho r} \quad (11)$$

what has again quite simple physical background. Equation (11) says us that the critical point must not be higher than the maximal height, to which water elevates because of capillarity. Beyond the critical point, the equation (8) changes to

$$Q_N^{(2)} = \frac{\pi}{8\eta}\frac{q^2R^4}{N}\frac{\frac{2\sigma}{r} - (2x-l)\rho g}{l} \quad (12)$$

and again using (9) we get

$$t_2 = \frac{8\eta N}{q^2R^2g^2\rho^2}\left[-\left(\frac{2\sigma}{r} - 2g\rho x\right)\ln\left(1 + \frac{g\rho r(l-x)}{2\sigma - \rho gxr}\right) + g\rho(l-x)\right] \quad (13)$$

for the second time. Than, the resulting time (time of the appearance of the first drop) is the sum:

$$T = t_1 + t_2 + T_{wet} \quad (14)$$

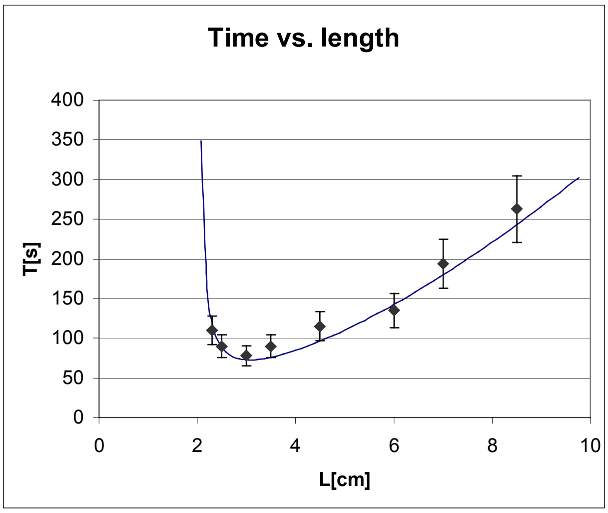
As mentioned, the evaporation is considered as in important phenomena in this problem. So we have also to count with it in our solution. We can express the evaporation flow:

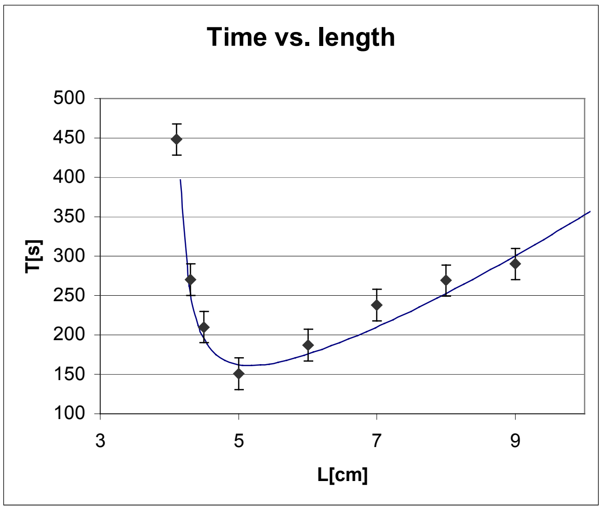
$$Q_{evap} = 2\pi Rl.C_{evap} \quad (15)$$

and change the equation (9) to the form

$$\left(Q_N(l) - Q_{evap}(l)\right)\mathbf{d}t = q\pi R^2\mathbf{d}l \quad (16)$$

Further solutions are similar, but results are even more complex, so we used only numerical solutions. Many experiments certify a good quality of the model, which is applicable in the most cases of fibers and threads.





Also experiments concerning some other parameters were done. It was shown, that the temperature of the water has an interesting influence on the time. Increasing the temperature within the room temperatures caused decreasing of the time, probably because the lower inner friction of hot water. Further increasing of the temperature led to the increase of the time, with the root in the abrupt increase of the evaporation flow.

Also humidity of the environment had some influence on the time, but it was rather simple and small.

Conclusions

First we try to enumerate the conditions, that has to be fulfilled for the first drop to appear:

- for the dry thread, **x**<the critical elevation height
- $h - x > \frac{2\sigma}{\rho g R}$, the gravitational force has to be strong enough to overwhelm the capillary pressure on the end of the thread
- evaporation through the surface of the thread has to be smaller then the flow in the fiber
- capillary effect has to work "in the right direction", the thread must by hydrophilic

Now we can discuss the influence of various parameters on the time itself:

- **L**, the total length of thread. By increasing of it the time increases (as seen on the right side of the graphs).
- **x**, the vertical distance between the water surface and the critical point. If it comes near to the critical height, the time is dramatically increasing
- **h**, the vertical distance from the critical point of the outer side of the vessel. Increase of this parameter brings a decrease of the time (as seen on the left side of the graphs. As **x** was held constant, increase of **L** mend also increase of **h**)
- material and structure of the thread had a big influence on the capillary effects. Some fibers did not get wet in its whole volume, or it took long time. It is very hard to quantify these parameters explicitly
- parameters of the water like temperature or salinity. These affect important constants used in equations like ρ, η or σ.
- humidity and temperature of the environment. These have big impact on the evaporation, but we did not made further investigation in this direction. All parameters are covered by one effective constant dependent on the fiber material and structure, water and environment together.

Epilogue

Probably the most obvious message from this presentation is, that it is extremely difficult to fit all the theory and experiments made by students in to the 12-minutes presentation. Here we spent 4 pages by describing it and still had to omit a lot of details and especially most of the experimental results. There exist many graphs with different dependencies, including the temperature, humidity and salinity, which had no chance to fit in this article.